Traveling-Wave Antenna Array (TWAA) with Multioctave Scan-Gain-Bandwidth

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***Abstract*—This paper presents the latest progress in demonstrating and characterizing the Traveling-Wave Antenna Array (TWAA) as a Multioctave Planar Phased Arrays (MPPA) capable of efficient wide-angle scan over multioctave bandwidth. (*Beam Steering Network (BSN) and T/R module are not included here.*) Four brassboard TWAA panels having 16×16 array elements were fabricated and tested for wide-angle scan up to 60° off broadside over 2-12 GHz using a True-Time Delay (TTD) BSN in a corporate feed configuration. Fairly good performance was exhibited using conventional far-field antenna range tests in anechoic chambers. This study appears to be the first time that an MPPA is demonstrated to be capable of multioctave bandwidth for efficient 60° scan in a full-fledged manner with direct calibration. The test data are in good agreements with computer simulation data generated by The Ohio State University. MPPA's state-of-the-art is reviewed, and relevant ambiguities and controversies are discussed.**

Keywords—array, phased array, planar array, broadband antenna, traveling wave antenna, traveling wave array, ultrawideband antenna, conformal array.

I. Introduction

Phased arrays are inherently much more complex and costly than other antennas, by at least an order of magnitude. To facilitate cost reduction in fabrication, installation, transport and maintenance, etc., they are mostly designed as an ensemble of planar panels. Since dawning of the Information Age around 1980, the need for increasingly broader spectral bandwidths and spatial coverage, as well as features, has led to vigorous research in pursuit of Multioctave Planar Phased Arrays (MPPA) capable of wide angle scan over a multioctave bandwidth. (One octave is 2:1 or ~67%.)

This paper presents the latest progress in the Traveling-Wave Antenna Array (TWAA) (*with Beam Steering Network (BSN) and T/R module excluded*). Supported by the Ohio State University (OSU) and Georgia Tech (GT), TWAA is one of several leading MPPAs under continuous development since 2007 [1]-[3]. In view of MPPA's many controversies, we will first review its history and the current state-of-the-art, and later address its ambiguities and misconceptions.

II. History of MPPA

It is worth noting that beam scan is implicit for a "phased array," and thus for MPPA by definition. For over 35 years, MPPA's key performance goals have been maximum angular scan to 60° off broadside with scan impedance matching of VSWR ≤ 2:1 over its claimed multioctave bandwidth.

It is also worth noting that nearly every MPPA credited Wheeler's Current Sheet Antenna (CSA) in 1965 [4] as its basic concept, while this author traced the embryonic root of TWAA to the concept of Walter [5]. Note also that the "Connected Array" (CA) has been employed by Hansen [6], Lee [7] and Neto and Cavallo [8] in the past two decades to mean MPPA.

The first major research in MPPA was conducted by Raytheon Company under a USAF contract (1981-1984), as reported by Grun and Pleva [9]. Its performance goal was active array scan to 60° half-cone angle, with VSWR<2, over 4.5-18.0 GHz. The research achieved an operating bandwidth of 10-14 GHz (33% bandwidth)—by relaxing SWR requirement to <3.

In 1999, Hemmi et al summarized continued research in MPPA since 1985 at Raytheon, which had greatly expanded to include leading phased array groups at Texas Instruments, Hughes, Motorola, etc. [10]. Supported by US NAVAIR and Raytheon internal funds, the studies had the same key performance goals as [9]. But they were focused on developing T/R module and BSN, apparently for narrowband or receive-only applications. For the array antenna *per se*, they employed Scan Element Gain (SEG) technique, which is only adequate for demonstrating Technology Readiness Level (TRL) up to 3.

MPPAs prior to 2000 universally employed the flared notch antenna as the array element, with its deadly high cross-polarization (X-Pol) problem. Only this author took a different approach, based on the first-generation Traveling-Wave Antenna (TWA) [5], [11]-[12]. *Thus, in 2000 every MPPA's demonstrated bandwidth was below 2:1 for scan to 60°.*

Toward the end of 1990s, wide-band Frequency Selective Surfaces (FSS) as low-RCS radome were successfully developed by the Munk team at OSU [13]. Munk and Harris Corporation engineers readily envisioned the potential of transforming Munk's FSS, which was basically a planar array of planar parallel dipoles without feeds, into a broadband wide-scan planar array antenna. Their design concept was named Tightly Coupled Array (TCA) [14]- [16].

The TCA inspired a flurry of publications, news releases, and patent applications for new MPPA concepts worldwide. Unfortunately, today most of them are still stagnating at the stage of computer simulation without credible experiments, or

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have fizzled. It was not until 2015 before valid prototype MPPAs began to be reported by Neto et al for the CA [8], [17] and by this author for the TWAA [18]-[21].

III. The State-of-the-Art of MPPA

The state-of-the-art of MPPA was discussed thoroughly by Neto and Cavallo in 2015 IEEE International Antenna Symposium [8] with further clarifications in [17]. They began with the observation that in 2003 there were in effect only "two competing families" in MPPA—the TCA family including Harris Corp. and OSU; and the Raytheon CA family including Lee, Hansen, and Neto, etc. They pointed out that by 2007 Raytheon's slot arrays reached 4:1 bandwidth but without beam scan.

Regarding the Tightly Coupled Dipole Arrays (TCDA) pursued by the OSU team under Volakis since 2008, e.g. [22]-[25], the comments in [8] were skeptical. Overall, TCDA served to differentiate the new design approach from the earlier TCA engineered by Harris Corp. and Munk of OSU. While there are impressive data showing multioctave bandwidths, at present TCDA is at TRL-4 at most. Thus, by the criteria set forth in [8], no reporting on MPPA at TRL⩾4 was known to the Neto group before their paper submission in January 2015.

Neto and Cavallo concluded that for efficient wide-angle scan "Connected Arrays" have cross-polarization (X-pol) higher than -10 dB due to problems in the feed; and that the "next step" was to try their conceptual design to reach a bandwidth "from 30% to 60% (1 octave)."

There are other clear and compelling evidences supporting the observations of the Neto team. For example, the plenary speech by Brookner of Raytheon in the Phased Array Symposiums in 2010 [26] stated that "Raytheon has demonstrated a notch radiating element that is capable of going from 1.8 GHz to 18 GHz, a 10:1 bandwidth. It can be dual polarized and scan to array with scan to 60°…", yet in the 2013 Symposium his plenary talk did not touch MPPA, showed only planar arrays with bandwidths much narrower than 2:1 [27].

Therefore, the assessment of the Neto group on the state-of-the-art of the MPPAs as of January 2015 [8] and [17] is accurate. Their ignoring immature MPPAs below TRL-4 is both necessary and desirable in view of the long-standing controversies in MPPAs [20]. Although the prohibitively high costs of fabricating and testing of a realistic MPPA breadboard are beyond the means of most research projects, yet perpetual stagnation at TRL-3 by computer simulation and SEG pattern tests is confusing and misleading to the community.

Therefore, a bird's-eye view on the state-of-the-art of MPPA, focused on practical issues and in particular between TWAA and other approaches, is presented in Table 1, with some clarifications later. Note that after presenting a TWAA prototype having a 2-12 GHz (or 6:1) instantaneous bandwidth [18], more comprehensive data have been published in two papers in Forum for Electromagnetic Research Methods and Application Technologies (FERMAT) [19]-[20] and a symposium paper [21].

TABLE I. The state-of-the-art of MPPA, with differentiations between TWAA and other approaches

Features	TWAA	Other MPPAs
Bandwidth and scan angle	Wide bandwidth and wide scan to 60° Bandwidth: 6:1 (2-12 GHz)	For efficient wide scan to 60° Bandwidth ≤ 2:1
Dissipative or exotic material (e.g., ferrite or metamaterial)	• Not used	• Often needed/used, thus low producibility • Large cost, weight & thickness
Substrates/ superstrates of special dielectric property	Not used (used only for structural support); thus lower cost, weight, thickness. Easily air cooled for high power!	Generally necessary; thus high cost, weight, thickness. Difficult to air cool, thus low power handling!

IV. The TWAA and Its Latest Performance Data

The TWAA had been discussed in [2]-[3] after its first public disclosure for patent application in 2007 filed by Wang Electro-Opto Corporation (WEO) [1]. Advanced development on TWAA began in 2014 and has been reported in [18]-[21]. In particular, independent tests on Model-A135-003 Serial #001 by Georgia Tech Research Institute (GTRI) in its Cobb-County anechoic chamber with satisfactory and confirming results [21] have established the viability of the TWAA as a leading MPPA technology, as summarized in Table I.

Fig. 1 shows photographs of the 2-12 GHz TWAA with 16×16 (256) elements, in front and back views, which is one of four models with identical designs on its front and back, having only minor differences in their feed structures.

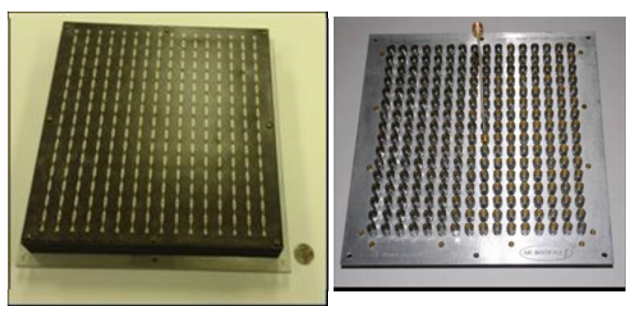


Fig. 1. Photograph of a TWAA in front and back views.

The design concept can be described in a unit-cell perspective, as discussed in [18]-[21]. The element radiators of the planar array are planar closely-coupled wideband bowtie dipoles, with widths s and b at the feed region and the two ends, respectively, as shown in Fig. 2, for a unit-cell of the array. To suppress grating lobes, the length l and width w of the unit cell are chosen to be $\lambda_h/2$, where λ_h is the free-space wavelength at the highest operating frequency. The superstrate and substrate are primarily for mechanical structure, though they can also be used to enhance performance and features.

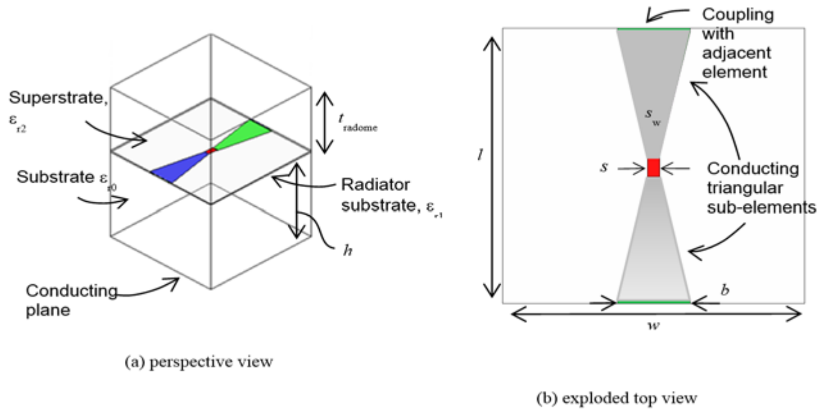


Fig. 2. (a) Unit cell of planar TWA array with planar bowtie dipole element, and (b) exploded top view of planar bowtie dipole element.

Fig. 3 depicts the cross-sectional view around a center element of a TWAA. Each dipole is capacitively coupled with adjacent dipoles at both ends (in the region of green color in Fig. 2 (b)) to facilitate Traveling-Wave (TW) propagation. The array center element radiator is fed by a transmission line which connects to an impedance-matched circuit leading to the feed network circuit contained inside a closed region below the ground plane. For systems integration, there is room of more than 1.27 cm in thickness below the ground plane (out of the 2.90-cm thickness) that can be used to accommodate T/R modules, BSN, etc.

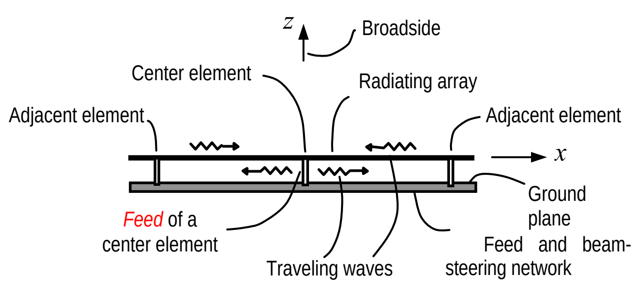


Fig. 3. Cross-sectional view around a center element of a TWAA.

As there was no available BSN covering 2-12 GHz, a True-Time-Delay (TTD) BSN as a corporate feed (with a single input/output) was developed in-house. (A TTD BSN generates time-shift, instead of phase shift, thus can also demonstrate performance for digital signals.) Discrete TTD lines for beam scan at 0º, ±30º, ±45º, and ±60º made of phase-matched semirigid coaxial cables, combined by three stages of 2-18 GHz power dividers, constitute the BSN, which is reciprocal for both transmit and receive. Such a BSN is very expensive and difficult to design, fabricate and set up, thus probably has not yet been made by others for large densely-packed arrays.

Fig. 4 shows the fully assembled array under far-field tests in WEO anechoic chamber, calibrated by standard gain horns. It includes the array panel (WEO model A135-003, SN 002) shown in Fig. 1 and the TTD BSN (the three banks behind it).

As can be seen, a mounting structure is installed on the antenna tower to support and interface with the array system on the platform. The array's phase center, located at the center of the front surface of the array panel, is aligned with the axis of azimuthal rotation of the antenna tower.

Fig. 5 shows measured and calculated antenna co-polarization (Co-pol) and cross-polarization (X-pol) gain data over 2-12 GHz at scan angles of 0º, 30º, 45º, and 60º for *E*-plane and *H*-plane scans, respectively, at the top and the bottom of the figure. The calculated gain patterns were generated by the Electro-Science Laboratory (ESL) of The Ohio State University (OSU), fairly independently. The calculated gain patterns were generated by multiplying the array factor and the computed Scan Element Gain (SEG) patterns of an infinite array with a moment-method solution using commercial software. Note that OSU did not generate simulation data for 60° scan, as explained in [18], and will be discussed in the next section.

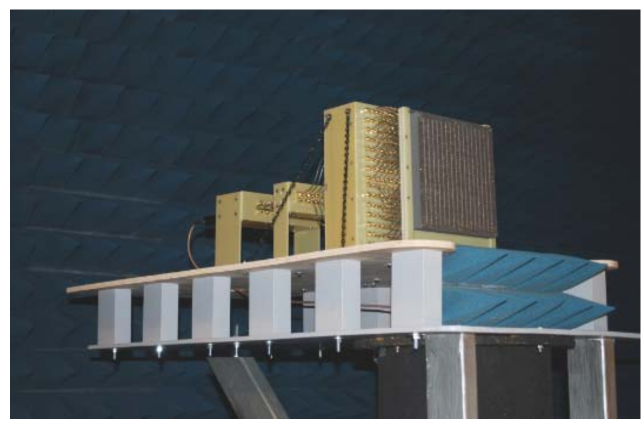
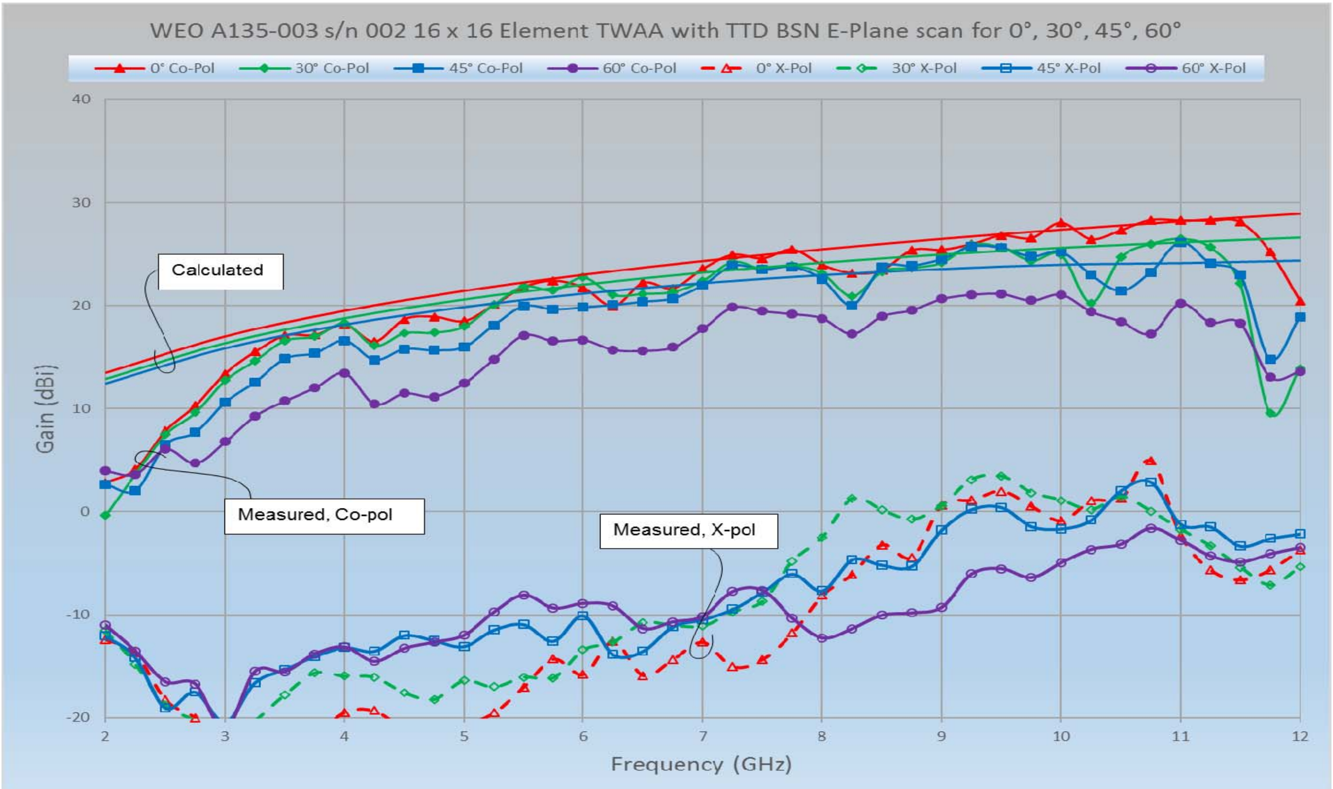


Fig. 4. A TWAA with three banks of TTD BSN being measured in WEO anechoic chamber.

That the measured gain, as compared with the theoretical gain, becomes lower with increasing scan angles is largely due to the mismatch between the scan impedance and the feed network, which can be improved by refining the feed network.

The X-pol for *E*-plane scan is consistently 20-30 dB below beam peak, except for frequencies below 2.5 GHz and above 11.5 GHz, where it is >10 dB below beam peak. For *H*-plane it is mostly >15 dB below beam peak, except for frequencies below 2.8 GHz and around 6.2 and 8.3 GHz. Major contributors for these higher X-pol problem are also in the feed region below the ground plane shown in Fig. 3, and thus can be dealt with by improving the feed network.

Fig. 6 and Fig. 7 show measured and simulated *E*-plane and *H*-plane gain patterns, respectively, for *E*-plane and *H*-plane scans at 0°, -30°, -45° and -60° off broadside at 2, 4, 8, and 12 GHz. The X-pol data are not shown here so that the quality of Co-pol pattern performance can be clearly displayed.



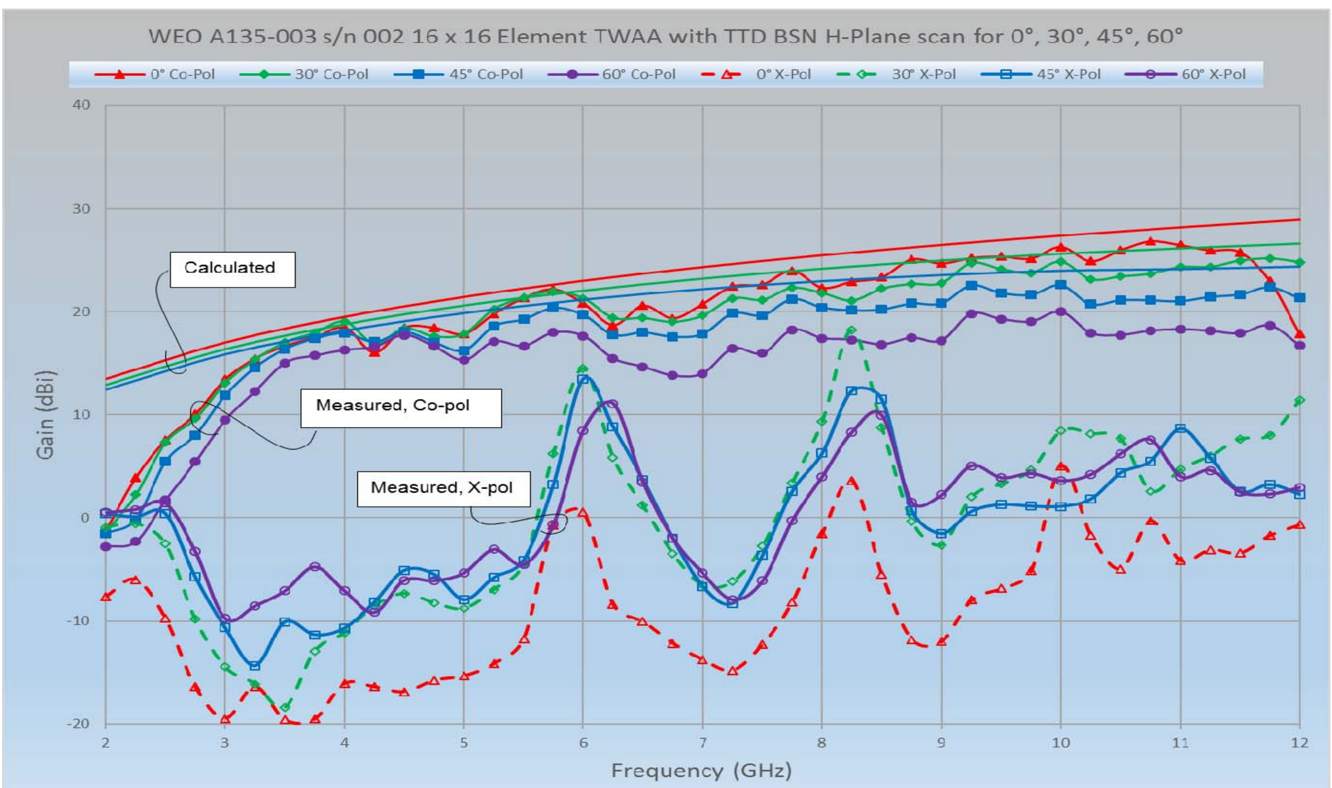


Fig. 5. Measured and calculated antenna co-polarization (Co-pol) and cross-polarization (X-pol) gain data over 2-12 GHz at scan angles of 0º, 30º, 45º, and 60º for *E*-plane scan (top) and *H*-plane scan (bottom).

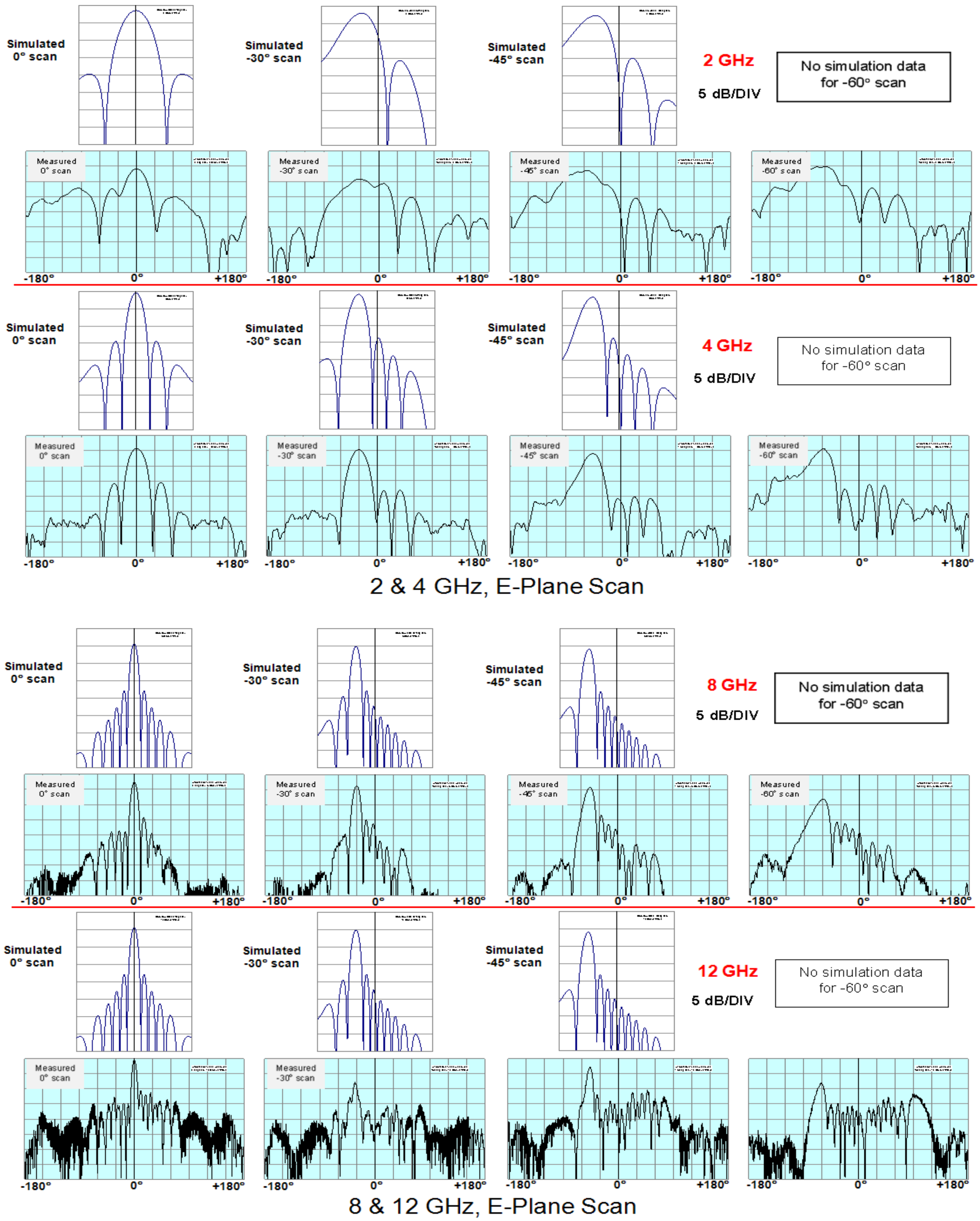


Fig. 6. Measured and simulated *E*-plane radiation patterns for *E*-Plane scan at 0°, -30°, -45° and -60° off broadside at 2, 4, 8, and 12 GHz.

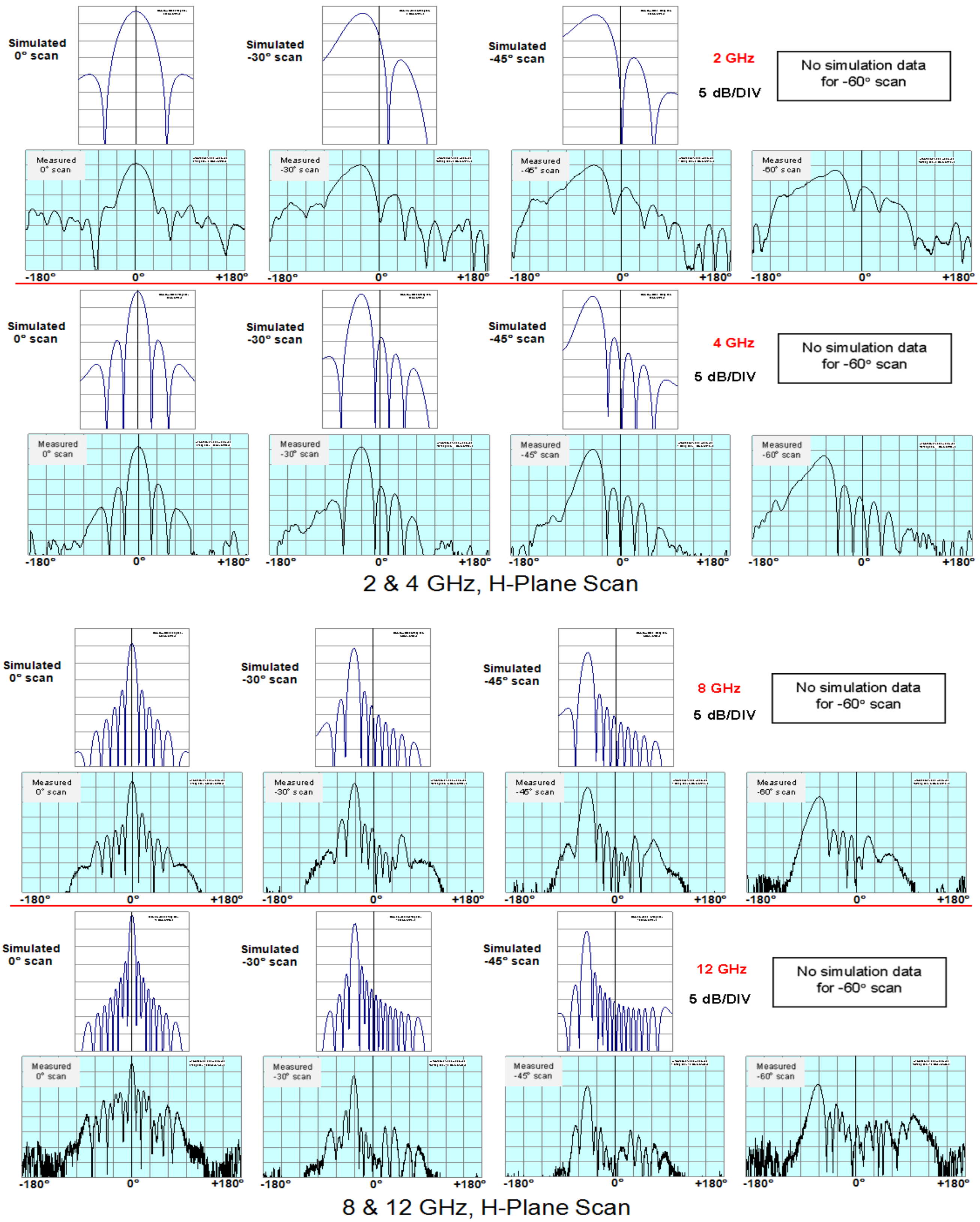


Fig. 7. Measured and simulated *H*-plane radiation patterns for *H*-Plane scan at 0°, -30°, -45° and -60° off broadside at 2, 4, 8, and 12 GHz.

V. FURTHER COMMENTS ON THE MEASURED AND COMPUTED DATA ON TWAA

Similar to earlier prototypes in [18]-[21], agreements between measured and computer-simulated patterns for the present prototype A135-003, SN002, are good except for scan angles beyond 45°, where no simulated data were provided by OSU. The measured patterns are again close to theoretical patterns based on classical array theory. The good results can be partially attributed to TWAA's robust performance observed in a simulation study of manufacturing tolerance performed by OSU earlier.

There were three prototype designs, which were essentially identical except for the feed region below the ground plane shown in Fig. 3. Four prototypes were fabricated and tested, with the third model, WEO A135-003, having two copies. Comparison of the measured data between the four prototypes indicates that the differences in performance are largely due to the differences in their feed networks. This finding shares the conclusion of Neto and Cavallo that they "worked on it 13 years to understand them and solve feeding mechanisms." [8].

As indicated in the preceding section, the minor deficiencies in the performance of the TWAA resulted largely from the mismatch between the scan impedance and the impedance of the input/output impedance of the feed network. First, impedance matching was carried out at 50 ohms between interfaces; it would be much easier to match at a higher impedance level in the feed network. Second, matching for the changing scan impedance can be achieved in the feed network.

The discrepancies between measured and computer-simulated data are also handicapped by the limitations of the commercial software and the computer used, as discussed in [18]. Indeed, based on the earlier observations by this author [28] and a review on the capabilities and applications of the latest software and hardware capabilities, the accuracy of the computation for wide scan angles is very questionable.

VI. SUBTLETIES, MISCONCEPTIONS, AND FUTURE OF MPPA

As pointed out earlier, the costs of fabrication and testing of a realistic breadboard (at least larger than 11×11 elements) are prohibitively high. Over the past 35 years, huge amounts of resources have been expended in MPPA research. From this author's perspective, many of them began with at least one fatal flaw in basic concept; we will try to expose the following subtleties and misconceptions in MPPAs.

Before proceeding, it is worth noting that, a long time ago, both Hansen [6] and Munk [14] made similar efforts in very lucid and strong languages. This author is moving one step closer to the root of the problems, even though their views have been expressed by him earlier [1]-[3] .

A. *The pitfall of formulating MPPA as a scattering problem*

Formulating the MPPA as a scattering problem has been a common practice in MPPA analysis and measurement. Obviously many were not aware of the fact that the equivalent circuit of an antenna as a scatterer cannot be represented by a Thevinin's or Norton's equivalent circuit (a common mistake in antenna textbooks). This subtle point had been discovered in early 1960s [29]-[30], and was employed by this author in antenna scattering problems [31]-[34].

For example, a major R&D thrust for an ultra-wideband reflect phased array using broadband planar spiral as array element antenna was sponsored by multiple agencies during 1973-1982. Started as classified projects, it was publicized with endorsements in a series of four articles in a major trade journal in 1975-1976. In 1981 this author noted a fatal flaw in the theory of its basic design approach and demonstrated that it could not be wideband as claimed [32]-[33].

B. *Wheeler's concept of Current Sheet Antenna (CSA)*

Most MPPA approaches credited their concept to Wheeler's Current Sheet Antenna (CSA) [4]. However, as pointed out by this author in column 2 of [1], Wheeler's array is impractical when a necessary ground plane is added.

In applying the CSA concept, it is worth noting that, for completeness, both electric and magnetic currents are required to represent the equivalent sources on the array surface. The X-pol problem in MPPAs as exposed by Neto and Cavallo [8] can be partially traced to a lack of completeness in its CSA formulation with scalar dyadic Green's functions.

Note that Wheeler implemented his CSA concept by treating each unit cell as a waveguide and then simplifying the problem by considering only the dominant propagating waveguide mode. This is an ingenious technique, which had also been used by this author to invent a "magnetic horn" with radiation pattern performance comparable to that of a well-designed corrugated horn [35]. Nevertheless, this is a narrowband concept not amenable to the MPPA.

C. *How to reenergize MPPA research*

As has been discussed, MPPA has been highly controversial, with booms and busts. Since about 2010, support for this research has rapidly declined. For example, the remarks that "Technology does not always equate to relevant capability" by a top government official in 2009 [36] epitomized profound disappointment and skepticism of governmental and industrial leaders during the past five years.

To reenergize MPPA research in view of the extreme high cost of developing a full MPPA system, a realistic approach is to apply the TWAA to a simple MPPA system, such as a passive, or solely transmit or receive, application.

VII. CONCLUDING REMARKS

The Traveling-Wave Antenna Array (TWAA) is shown to be a Multioctave Planar Phased Arrays (MPPA) capable of efficient wide scan with low cross polarization. The claim of wide-angle scan up to 60° off broadside over 2-12 GHz (a 6:1 instantaneous bandwidth) is based on good measured and computed data, reinforced by potential performance improvements that can be achieved at the feed network.

With four prototypes using standard commercial PCB fabrication processes and commercial-off-the-shelf (COTS) parts and materials, TWAA has reached TRL-7 and MRL-7.

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